

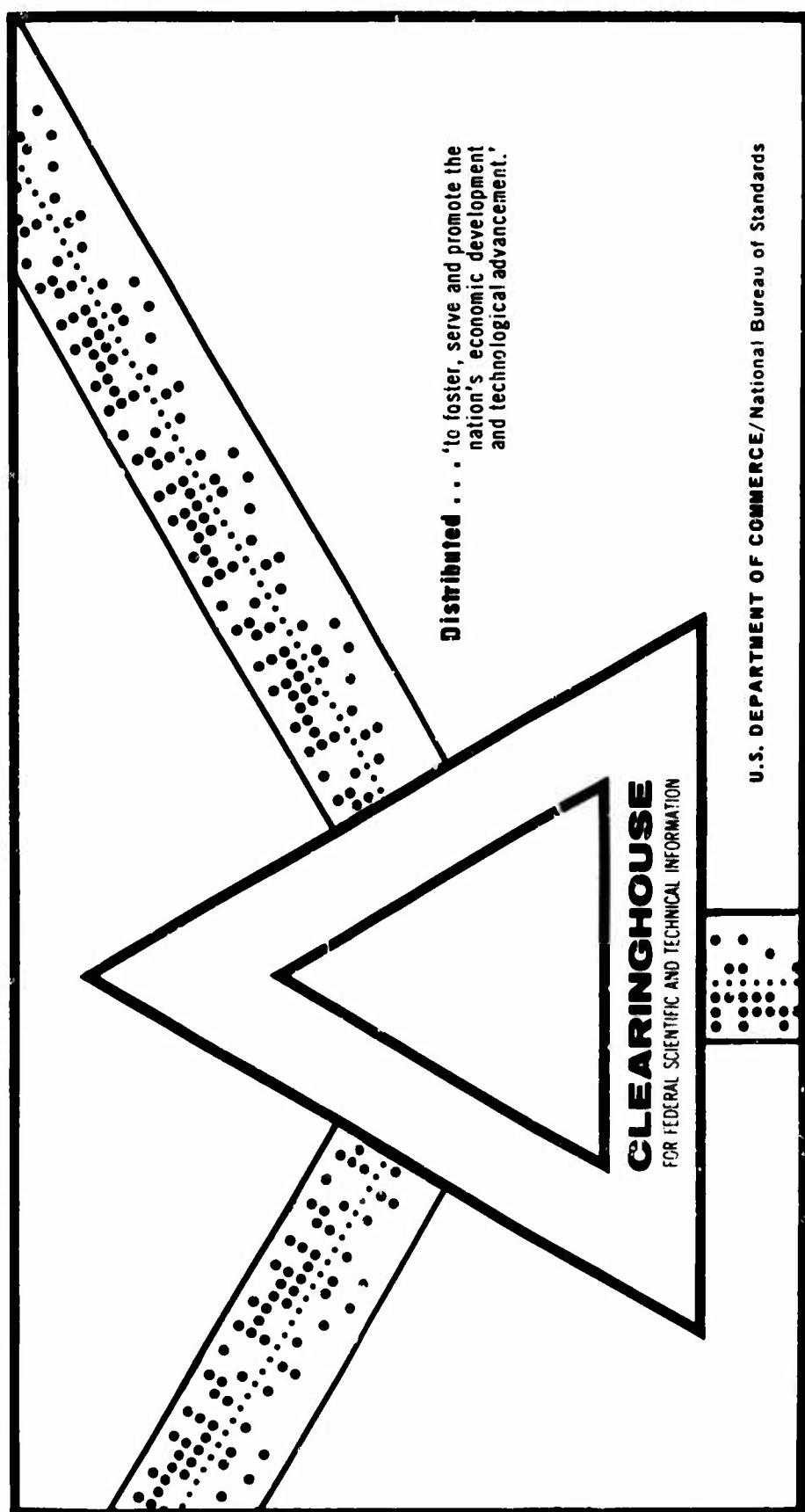
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MINIMUM SAMPLE SIZES FOR COMPARISONS USING CONTINUOUS VARIABLES

Lila E. Massa

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8 October 1969



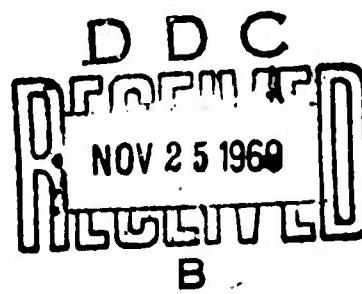
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AD 696966

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PREPARED BY

**RESEARCH AND DEVELOPMENT DEPARTMENT  
NAVAL AMMUNITION DEPOT, CRANE, INDIANA**

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**ABSTRACT**

When comparing the means from two populations, minimum sample sizes to detect a given difference may be determined. Presented are tables which may be used when unequal as well as equal sample sizes are desired.

## INTRODUCTION

Frequently experimenters make comparisons of two population means based on random samples drawn from each of the populations. Too many times, a real and possibly important difference has occurred but the experiment was not sufficiently sensitive to detect the difference. To correct this problem, in the design stage, not only  $\alpha$  (the probability of asserting that a difference exists, when actually there is no difference) but also  $\beta$  (the probability of claiming no difference exists when actually the means are different) for a given alternative hypothesis can be determined. The chosen levels can be preserved by taking adequate sample sizes.

An example of this type of problem is the following: Illuminating flares are being manufactured with a load pressure of 10,000 pounds per square inch (psi). It is hypothesized that a load pressure of 20,000 psi will increase the efficiency of the flare. If the efficiency of the flare using the new technique is 4,000 candlepower-seconds/gram greater than the efficiency using the old technique, the experimenter would like to have a high degree of assurance that a significant result will be obtained in the experiment. Due to the cost of the equipment required

to make a change,  $\alpha$  is important.  $\beta$  is also important because an increased efficiency is desirable. A decision is made to set  $\alpha = .05$  and  $\beta = .05$ . The question to be asked now is for  $\alpha = .05$ , what are the minimum sample sizes so that .05 is the probability of accepting the hypothesis of no difference when actually there is a difference of at least 4,000 psi.

Tables are published in several statistics books which give the required sample sizes; however, these sets of tables are only for equal sample sizes. The purpose of this article is to present tables for unequal as well as equal sample sizes.

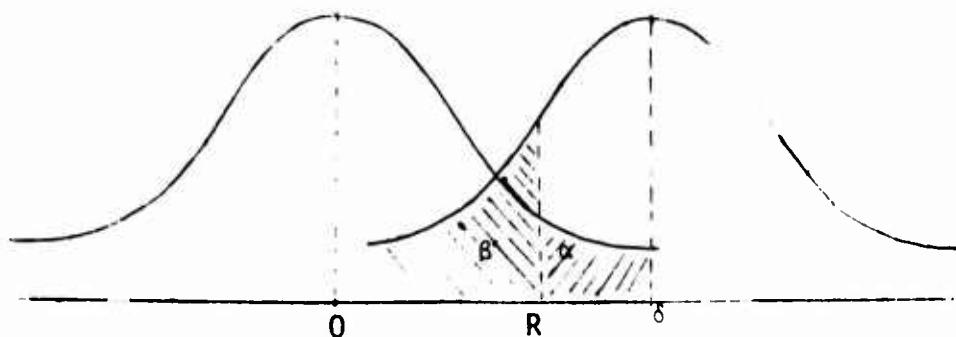
MATHEMATICAL DISCUSSION

The problem of finding the minimum number of observations needed is that of finding  $n_1$  and  $n_2$  so that the distribution of  $\bar{x}_1 - \bar{x}_2$ , when  $\mu_1 - \mu_2 = 0$  and  $\mu_1 - \mu_2 = \delta$ , will overlap a critical value  $R$  by at most  $\alpha$  and  $\beta$  respectively. If the two populations are normal and have equal but unknown variances, then the proper statistic to use is the 't' statistic.

$$t_v = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad v = n_1 + n_2 - 2 \quad (1)$$

where  $S_p = \sqrt{\frac{(n_1-1) S_1^2 + (n_2-1) S_2^2}{n_1 + n_2 - 2}}$

Consider the distribution of  $(\bar{x}_1 - \bar{x}_2)$



The curve on the left is under the null hypothesis  $\mu_1 - \mu_2 = 0$  while the curve on the right is under the alternative hypothesis  $\mu_1 - \mu_2 = \delta$ .

Under the null hypothesis,

$$t_{(1-\alpha, v)} = \frac{R-0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad (2)$$

Solve for R in equation (2)

$$R = t_{(1-\alpha, v)} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad (3)$$

Under the alternative hypothesis

$$t_{(\beta, v)} = \frac{R-\delta}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad (4)$$

Solve for  $R - \delta$  in equation (4)

$$R - \delta = t_{(\beta, v)} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad (5)$$

Subtract (5) from (3)

$$\begin{aligned} R - (R-\delta) &= S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} t_{(1-\alpha, v)} \\ &\quad - S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} t_{(\beta, v)} \end{aligned} \quad (6)$$

By simplification

$$\delta = (t_{(1-\alpha, v)} - t_{(\beta, v)}) S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad (7)$$

Divide (7) by  $S_p$  and substitute

$$-t_{(\beta, v)} = t_{(1-\beta, v)}$$

$$\frac{\delta}{S_p} = (t_{(1-\alpha, v)} + t_{(1-\beta, v)}) \sqrt{1/n_1 + 1/n_2} \quad (8)$$

$$\text{Let } D = \delta/S_p$$

$$D = (t_{(1-\alpha, v)} + t_{(1-\beta, v)}) \sqrt{1/n_1 + 1/n_2} \quad (9)$$

The computations of the above equation are not difficult; however, it is time-consuming to find minimum values of  $n_1$  and  $n_2$  without a table especially since a trial-and-error procedure is involved. Suppose an experimenter has estimated  $S_p$  and has selected values of  $\delta$ ,  $\alpha$ , and  $\beta$ . The trial-and-error procedure is as follows:

- (1) Select a good guess for  $n_1$  and  $n_2$ .
- (2) From a "t" - table find  $t_{(1-\alpha, v)}$  and  $t_{(1-\beta, v)}$ .
- (3) Compute this  $D$  value (call it  $D_{\text{Min}}$ ).
- (4) Repeat steps 1, 2, 3 until the given  $D \geq D_{\text{Min}}$  and a decrease in  $n_1$  or  $n_2$  would result in  $D < D_{\text{Min}}$ .

An example of this process follows for a one-tailed test:

Suppose  $\delta = 14.5$ ,  $S_p = 10$ ,  $\alpha = .05$ ,  $\beta = .05$ .

Then  $D = 1.45$

Guess:  $n_1 = n_2 = 10$

By substitution into (9)

$$(t_{(.95, 18)} + t_{(.95, 18)}) \sqrt{1/10 + 1/10} = 1.55$$

This result implies that for equal sample sizes  $n_1 = n_2 = 10$ , the difference it is important to detect will be at least 15.5 for  $\alpha = .05$  and  $\beta = .05$ . To detect a difference of 14.5 for the same  $\alpha$  and  $\beta$ , larger sample sizes are needed.

Guess:  $n_1 = n_2 = 12$

Calculating  $D_{\text{Min}}$ , the result is 1.40. Thus a difference of at least 14.0 will be declared for  $\alpha = .05$  and  $\beta = .05$ . This sample size is satisfactory but a check must be made to see if a smaller sample size will do just as well.

Check:  $n_1 = n_2 = 11$

For this sample size,  $D_{\text{Min}} = 1.47$  which is too large. The correct answer for equal sample sizes is 12.

Tables are much quicker to use than the above process. The tables in the appendix present values of  $D_{\text{Min}}$  for varying combinations of  $\alpha$  and  $\beta$  as well as  $n_1$  and  $n_2$ . Values up to

80 are tabled for  $n_1$  and  $n_2$ . These values were calculated from equation (9) by means of a computer. The resulting values were then rounded off to the second decimal place. With these tables, sample sizes can be determined with a minimum amount of computation and time spent.

#### PROCEDURE FOR USING TABLES

- (1) Estimate  $S_p$ , the standard deviation
- (2) Specify  $\delta$ , the difference between means that it is important to detect.
- (3) Select  $\alpha$  and  $\beta$ .
- (4) Compute  $D$  where  $D = \delta/S_p$
- (5) In the appropriate table, find  $n_1$  and  $n_2$  as follows:

- a. for  $n_1 = n_2$

Using the diagonal elements for  $D_{Min}$ , find the minimum  $n_1$  and  $n_2$  such that the calculated  $D$  is greater than or equal to the tabled value of  $D_{Min}$ .

- b. for  $n_1$  fixed

If  $n_1$  is fixed, find the minimum  $n_2$  such that the calculated  $D$  is greater than or equal to the tabled value of  $D_{Min}$  in the column headed by  $n_1$ .

c.. for  $n_1 \neq n_2$

Various combinations of  $n_1$  and  $n_2$  are presented.

It should be noted that the total of both samples ( $n_1$  and  $n_2$ ) for a given value of  $D_{Min}$  is minimized when  $n_1 = n_2$  (due to the discrete nature of  $n_1$  and  $n_2$ , a few minor exceptions exist). The criterion here is to choose the largest tabled value of  $D$  which is less than the observed value of  $D$ . When costs per unit of  $n_1$  and  $n_2$  are unequal, choose the combination of  $n_1$  and  $n_2$  which satisfy the condition given above and which minimize the cost.

#### EXAMPLES

(1) ( $n_1 = n_2$ ), one-tailed test

Given:  $S_p = 10$ ,  $\delta = 18$ ,  $\alpha = .025$ ,  $\beta = .01$

Find: minimum  $n_1$  and  $n_2$  required to detect a difference of 18

Compute:  $D = \delta/S_p = 18/10 = 1.80$

Consult table 11

for  $n_1 = n_2 = 12$ ,  $D_{Min} = 1.87$

for  $n_1 = n_2 = 13$ ,  $D_{Min} = 1.79$

∴ the required sample sizes are  $n_1 = n_2 = 13$

(2) ( $n_1$  fixed), one-tailed test

Given:  $S_p = 10$ ,  $\delta = 18$ ,  $\alpha = .025$ ,  $\beta = .01$ ,  $n_1 = 10$

Find: minimum  $n_2$  required to detect a difference of 18

Compute:  $D = \delta/S_p = 18/10 = 1.8$

Consult table 11

for  $n_1 = 10$  and  $n_2 = 17$ ,  $D_{Min} = 1.81$

for  $n_1 = 10$  and  $n_2 = 18$ ,  $D_{Min} = 1.79$

∴ the required sample sizes are  $n_1 = 10$  and  $n_2 = 18$

(3) ( $n_1 \neq n_2$ , neither fixed), one-tailed test

Given: The cost per unit of  $n_1$  is 5 times the cost of  $n_2$

$S_p = 10$ ,  $\delta = 18$ ,  $\alpha = .025$ ,  $\beta = .01$

Find:  $n_1$  and  $n_2$  such that the total cost  $(5n_1 + n_2)$  is minimized and a difference of 18 will be detected

Compute:  $D = \delta/S_p = 18/10 = 1.8$

Consult table 11 for various values of  $n_1$ . List the smallest value of  $n_2$  which would detect the difference of 18. The combinations and costs are as follows:

<u><math>n_1</math></u>	<u><math>n_2</math></u>	Cost <u><math>5n_1 + n_2</math></u>
7	45	80
8	28	68
9	21	66
10	18	68
11	16	71
12	14	74
13	13	78
14	12	82

$\therefore$  The minimum cost is achieved for  $n_1 = 9$  and  $n_2 = 21$

#### OTHER APPLICATIONS

These tables may also be used to determine the minimum accepted difference ( $\delta$ ) for a given  $S_p$ ,  $n_1$ ,  $n_2$ ,  $\alpha$  and  $\beta$ .

(4) ( $n_1 \neq n_2$ ), one-tailed test

Given:  $n_1 = 45$ ,  $n_2 = 22$ ,  $S_p = 10$ ,  $\alpha = .025$ ,  $\beta = .01$

Find: the smallest value of  $\delta$  for  $\alpha$  and  $\beta$  as above.

Consult table 11

$D_{Min} = 1.14$  (i.e., 1.14 is the smallest  $D$  for  $n_1 = 45$ ,  $n_2 = 22$  where a difference of  $\delta$  is detected while  $\alpha = .025$ ,  $\beta = .01$ )

$$D_{Min} = \delta/S_p$$

$$1.14 = \delta/10$$

$\therefore \delta = 11.4$  would yield significant results when  
 $\alpha = .025$  and  $\beta = .01$

ACKNOWLEDGEMENTS

The author wishes to thank Jerry Kemp and Ralph Chipman for their assistance and David Kemp for developing the computer program.

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APPENDIX A

TABLE I

		Single-sided test $\alpha = .005$										Double-sided test $\alpha = .01$																															
$n_1$	$n_2$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	35	40	45	50	60	70	80						
2	26.89	9.48	7.23	6.39	5.59	5.21	4.94	4.68	4.43	4.18	4.05	4.03	4.23	4.18	4.05	4.05	4.05	4.05	3.98	3.96	3.93	3.91	3.89	3.87	3.86	3.84	3.83	3.81	3.80	3.79	3.69	3.66	3.63	3.61	3.59								
3	9.48	6.62	5.65	5.00	4.59	4.31	4.11	3.96	3.83	3.76	3.65	3.59	3.53	3.48	3.44	3.40	3.37	3.34	3.31	3.29	3.27	3.25	3.23	3.21	3.20	3.18	3.17	3.16	3.15	3.10	3.07	3.04	3.02	2.99	2.96	2.95							
4	7.23	5.65	4.84	4.26	4.04	3.81	3.63	3.50	3.39	3.31	3.23	3.17	3.12	3.08	3.04	3.00	2.97	2.94	2.92	2.90	2.87	2.86	2.84	2.82	2.81	2.80	2.78	2.77	2.77	2.72	2.69	2.66	2.64	2.61	2.58	2.57							
5	6.39	5.00	4.36	3.95	3.68	3.47	3.32	3.20	3.10	3.02	2.95	2.90	2.85	2.80	2.77	2.73	2.70	2.68	2.65	2.63	2.61	2.59	2.56	2.55	2.54	2.53	2.52	2.52	2.47	2.47	2.43	2.38	2.35	2.33	2.31								
6	5.99	4.79	4.04	3.68	3.43	3.24	3.10	2.98	2.89	2.82	2.75	2.70	2.65	2.61	2.57	2.54	2.51	2.49	2.46	2.44	2.42	2.40	2.39	2.37	2.36	2.35	2.34	2.34	2.33	2.30	2.28	2.26	2.24	2.21	2.19	2.16	2.14	2.12	2.10	2.08			
7	5.21	4.31	3.61	3.47	3.24	3.07	2.93	2.82	2.73	2.66	2.60	2.55	2.50	2.46	2.42	2.39	2.36	2.34	2.32	2.29	2.26	2.24	2.23	2.22	2.21	2.20	2.17	2.17	2.17	2.17	2.17	2.17	2.17	2.17	2.17	2.17	2.17	2.17					
8	4.94	4.11	3.63	3.32	3.10	2.93	2.80	2.70	2.61	2.54	2.48	2.43	2.38	2.34	2.31	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.13	2.12	2.11	2.10	2.09	2.08	2.05	2.01	1.98	1.95	1.93	1.90	1.88	1.86	1.85						
9	4.75	3.96	3.50	3.29	2.98	2.82	2.70	2.59	2.51	2.44	2.36	2.33	2.29	2.25	2.21	2.18	2.15	2.13	2.10	2.08	2.06	2.05	2.04	2.02	2.01	2.00	1.99	1.97	1.96	1.92	1.90	1.88	1.86	1.84	1.82	1.80	1.78	1.76					
10	4.60	3.63	3.39	3.10	2.90	2.73	2.61	2.51	2.43	2.36	2.30	2.25	2.21	2.17	2.13	2.10	2.07	2.05	2.03	2.00	1.99	1.97	1.96	1.94	1.91	1.90	1.88	1.86	1.85	1.84	1.83	1.82	1.81	1.80	1.79	1.78	1.77	1.77	1.76	1.75			
11	4.48	3.76	3.31	3.02	2.92	2.82	2.66	2.54	2.44	2.36	2.29	2.23	2.18	2.14	2.10	2.06	2.03	2.01	1.98	1.96	1.94	1.92	1.91	1.90	1.88	1.86	1.85	1.84	1.83	1.82	1.81	1.80	1.79	1.78	1.77	1.76	1.75	1.74					
12	4.38	3.65	3.23	2.95	2.75	2.60	2.48	2.39	2.30	2.23	2.17	2.12	2.08	2.04	2.01	1.98	1.95	1.92	1.90	1.88	1.87	1.85	1.84	1.83	1.82	1.81	1.80	1.79	1.78	1.77	1.77	1.76	1.75	1.75	1.75	1.75	1.75	1.75					
13	4.30	3.79	3.37	3.09	2.90	2.70	2.55	2.43	2.33	2.25	2.18	2.12	2.07	2.03	1.99	1.96	1.93	1.90	1.87	1.86	1.84	1.82	1.81	1.78	1.76	1.75	1.74	1.73	1.72	1.71	1.70	1.69	1.68	1.67	1.66	1.65	1.64						
14	4.23	3.53	3.32	3.05	2.85	2.65	2.50	2.38	2.29	2.21	2.14	2.08	2.03	1.99	1.95	1.91	1.88	1.86	1.83	1.81	1.80	1.78	1.75	1.74	1.72	1.71	1.70	1.68	1.67	1.66	1.65	1.64	1.63	1.62	1.61	1.60	1.59						
15	4.18	3.48	3.08	2.86	2.61	2.46	2.34	2.25	2.17	2.10	2.04	1.99	1.95	1.91	1.88	1.84	1.82	1.80	1.78	1.76	1.73	1.71	1.70	1.69	1.67	1.65	1.64	1.63	1.62	1.61	1.60	1.59	1.58	1.57	1.56	1.55	1.54	1.53	1.52	1.51	1.50		
16	4.13	3.44	3.04	2.77	2.57	2.42	2.31	2.21	2.13	2.06	2.01	1.96	1.91	1.88	1.84	1.81	1.79	1.77	1.75	1.73	1.71	1.70	1.68	1.67	1.65	1.63	1.62	1.61	1.60	1.59	1.58	1.57	1.56	1.55	1.54	1.53	1.52	1.51	1.50				
17	4.09	3.40	3.07	2.79	2.56	2.39	2.18	2.10	2.03	1.98	1.93	1.88	1.84	1.80	1.76	1.72	1.70	1.68	1.67	1.65	1.64	1.63	1.61	1.60	1.59	1.57	1.55	1.54	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39
18	4.05	3.37	2.97	2.70	2.51	2.36	2.26	2.15	2.07	2.01	1.95	1.90	1.86	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62	1.60	1.58	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39		
19	4.03	3.34	2.84	2.68	2.49	2.34	2.22	2.13	2.05	1.98	1.92	1.87	1.83	1.80	1.77	1.74	1.70	1.67	1.65	1.63	1.62	1.59	1.57	1.55	1.53	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39					
20	3.98	3.31	2.92	2.65	2.46	2.32	2.20	2.10	2.03	1.96	1.90	1.86	1.81	1.78	1.75	1.70	1.68	1.65	1.63	1.61	1.58	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33					
21	3.96	3.29	2.90	2.63	2.44	2.29	2.18	2.08	2.00	1.94	1.88	1.80	1.76	1.71	1.68	1.66	1.63	1.61	1.58	1.56	1.53	1.52	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34					
22	3.93	3.27	2.87	2.61	2.42	2.35	2.16	2.06	1.99	1.92	1.87	1.82	1.78	1.73	1.70	1.67	1.64	1.62	1.59	1.56	1.53	1.51	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33				
23	3.92	3.25	2.86	2.69	2.40	2.26	2.14	2.05	1.97	1.91	1.85	1.81	1.75	1.71	1.68	1.65	1.62	1.59	1.57	1.55	1.53	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.31			
24	3.89	3.23	2.84	2.68	2.43	2.28	2.14	2.04	1.96	1.90	1.84	1.78	1.74	1.70	1.67	1.64	1.60	1.57	1.55	1.53	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.31				
25	3.87	3.21	2.82	2.64	2.45	2.27	2.13	2.02	1.95	1.88	1.83	1.76	1.72	1.69	1.65	1.61	1.58	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.30	1.29					
26	3.86	3.20	2.81	2.63	2.43	2.26	2.11	2.01	1.94	1.86	1.80	1.75	1.71	1.67	1.63	1.60	1.57	1.55	1.52	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.30	1.29					
27	3.84	3.18	2.80	2.58	2.35	2.21	2.10	2.00	1.91	1.85	1.79	1.74	1.70	1.65	1.62	1.59	1.56	1.54	1.51	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.30					
28	3.83																																										

TABLE 2

Single-sided test  $\alpha = .005$      $\beta = .05$

$$\text{Value of } D = \delta/S_D$$

TABLE 3

Single-sided test = .005       $\beta = .10$   
 Double-sided test = .01

Value of D =  $s/S_p$

TABLE 4

	Single-sided test $\alpha = .005$												$\beta = .20$																													
	Double-sided test $\alpha = .01$																																									
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	40	45	50	60	70	80						
2	10.69	6.32	4.80	4.16	3.77	3.52	3.36	3.23	3.14	3.06	3.00	2.97	2.91	2.87	2.84	2.81	2.79	2.77	2.75	2.7	2.72	2.70	2.69	2.68	2.67	2.66	2.65	2.63	2.60	2.58	2.56	2.52	2.51	2.49								
3	6.22	4.53	3.78	3.37	3.11	2.93	2.80	2.70	2.62	2.55	2.51	2.46	2.43	2.39	2.37	2.34	2.32	2.30	2.28	2.27	2.26	2.23	2.22	2.21	2.20	2.19	2.18	2.15	2.13	2.11	2.10	2.08	2.06	2.05								
4	4.69	3.78	3.05	2.65	2.76	2.59	2.48	2.39	2.32	2.27	2.22	2.15	2.15	2.12	2.05	2.07	2.05	2.03	2.01	2.00	1.99	1.97	1.96	1.95	1.93	1.92	1.91	1.90	1.89	1.88	1.87	1.86	1.85	1.83	1.81	1.80	1.79					
5	4.24	3.37	2.75	2.48	2.50	2.37	2.27	2.19	2.13	2.07	2.03	1.99	1.96	1.93	1.91	1.88	1.87	1.85	1.83	1.82	1.80	1.79	1.78	1.77	1.75	1.75	1.74	1.73	1.72	1.71	1.69	1.67	1.65	1.63	1.62	1.61						
6	3.77	3.11	2.76	2.50	2.24	2.22	2.05	1.99	1.94	1.86	1.82	1.69	1.77	1.75	1.72	1.70	1.69	1.66	1.67	1.64	1.65	1.68	1.63	1.62	1.61	1.60	1.59	1.58	1.57	1.56	1.55	1.54	1.52	1.50	1.49	1.48						
7	3.52	2.93	2.59	2.37	2.22	2.10	2.01	1.96	1.83	1.79	1.75	1.72	1.70	1.67	1.65	1.63	1.62	1.60	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.40	1.39	1.37				
8	3.36	2.80	2.48	2.27	2.12	2.01	1.92	1.85	1.82	1.78	1.72	1.67	1.66	1.62	1.59	1.57	1.55	1.54	1.52	1.51	1.49	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.30	1.29		
9	3.23	2.70	2.39	2.19	2.06	1.94	1.85	1.78	1.77	1.69	1.64	1.61	1.58	1.55	1.52	1.51	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.25			
10	3.14	2.62	2.32	2.13	1.99	1.88	1.80	1.73	1.67	1.63	1.59	1.55	1.52	1.47	1.45	1.42	1.40	1.39	1.37	1.37	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.19			
11	3.06	2.56	2.27	2.07	1.96	1.85	1.75	1.68	1.63	1.59	1.54	1.51	1.48	1.45	1.43	1.41	1.39	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.19	1.17	1.15	1.14	1.13	1.12		
12	3.00	2.51	2.22	2.07	1.99	1.89	1.79	1.71	1.64	1.59	1.54	1.50	1.47	1.44	1.41	1.39	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.19	1.18	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.08
13	2.95	2.46	2.18	1.99	1.86	1.75	1.67	1.57	1.51	1.45	1.41	1.37	1.33	1.29	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1.00	1.00			
14	2.91	2.43	2.15	1.96	1.82	1.72	1.64	1.58	1.52	1.48	1.44	1.40	1.37	1.33	1.30	1.28	1.25	1.22	1.20	1.18	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
15	2.87	2.39	2.12	1.97	1.80	1.70	1.62	1.55	1.51	1.45	1.42	1.37	1.33	1.29	1.25	1.22	1.20	1.18	1.15	1.13	1.11	1.09	1.07	1.05	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89					
16	2.84	2.37	2.09	1.91	1.77	1.67	1.59	1.53	1.47	1.43	1.39	1.35	1.31	1.28	1.25	1.22	1.20	1.18	1.15	1.13	1.11	1.09	1.07	1.05	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89					
17	2.81	2.34	2.07	1.89	1.75	1.65	1.57	1.51	1.45	1.41	1.37	1.33	1.29	1.26	1.23	1.20	1.18	1.15	1.13	1.11	1.09	1.07	1.05	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89						
18	2.79	2.32	2.05	1.87	1.73	1.63	1.55	1.49	1.43	1.39	1.35	1.31	1.28	1.24	1.21	1.18	1.15	1.12	1.10	1.08	1.05	1.03	1.01	0.99	0.97	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	
19	2.77	2.30	2.03	1.85	1.72	1.62	1.54	1.47	1.42	1.37	1.33	1.29	1.25	1.22	1.19	1.16	1.13	1.10	1.07	1.04	1.01	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75			
20	2.75	2.28	2.01	1.83	1.70	1.60	1.52	1.46	1.36	1.32	1.28	1.26	1.22	1.20	1.18	1.15	1.12	1.10	1.07	1.04	1.01	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75			
21	2.73	2.27	2.00	1.82	1.69	1.59	1.51	1.44	1.39	1.34	1.30	1.27	1.24	1.22	1.19	1.17	1.14	1.12	1.10	1.08	1.05	1.03	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75	
22	2.72	2.26	1.99	1.80	1.67	1.57	1.49	1.43	1.37	1.33	1.29	1.25	1.22	1.19	1.16	1.13	1.10	1.08	1.05	1.02	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75			
23	2.70	2.24	1.97	1.86	1.74	1.66	1.56	1.48	1.42	1.37	1.32	1.28	1.25	1.22	1.19	1.16	1.13	1.10	1.08	1.05	1.03	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75		
24	2.69	2.23	1.96	1.76	1.65	1.55	1.45	1.39	1.33	1.27	1.23	1.20	1.17	1.14	1.11	1.08	1.05	1.02	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.74				
25	2.68	2.22	1.95	1.77	1.64	1.54	1.46	1.39	1.34	1.28	1.24	1.21	1.18	1.15	1.12	1.09	1.06	1.03	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.74				
26	2.67	2.21	1.94	1.76	1.63	1.53	1.46	1.39	1.34	1.28	1.24	1.21	1.18	1.15	1.12	1.09	1.06	1.03	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.74				
27	2.66	2.20	1.93	1.75	1.63	1.53	1.45	1.39	1.34	1.28	1.24	1.21	1.18	1.1																												

TABLE 5

Single-sided test  $\alpha = .005$      $\beta = .50$

Double-sided test $\alpha = .01$																																			
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	35	40	45	50	60	70	80	
2	9.49	8.53	7.99	7.37	6.73	6.23	5.65	5.04	4.45	3.99	3.54	3.13	2.73	2.35	2.07	1.81	1.63	1.45	1.27	1.11	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
3	5.16	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	5.08	
4	3.99	3.68	3.25	2.77	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.89	1.87	1.85	1.76	1.72	1.69	1.66	1.63	1.61	1.59	1.57	1.56	1.54	1.53	1.52	1.50	1.49	1.48	1.47	1.47	1.46	1.46	1.46	
5	3.37	2.71	2.35	2.17	2.05	1.94	1.87	1.81	1.77	1.72	1.69	1.65	1.63	1.61	1.59	1.57	1.56	1.54	1.52	1.51	1.50	1.49	1.48	1.47	1.47	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	1.46	
6	3.07	2.47	2.17	1.97	1.86	1.77	1.70	1.65	1.61	1.57	1.54	1.51	1.49	1.47	1.45	1.42	1.42	1.40	1.39	1.36	1.35	1.35	1.34	1.33	1.33	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	
7	2.58	2.06	1.86	1.77	1.69	1.63	1.56	1.50	1.45	1.41	1.38	1.35	1.32	1.30	1.28	1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.18	1.18	1.17	1.17	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16	
8	2.46	2.02	1.84	1.77	1.65	1.56	1.49	1.43	1.39	1.35	1.31	1.29	1.26	1.24	1.22	1.20	1.19	1.17	1.16	1.15	1.14	1.13	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12		
9	2.34	2.11	1.87	1.70	1.59	1.50	1.43	1.38	1.33	1.29	1.25	1.21	1.19	1.17	1.15	1.14	1.14	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	
10	2.45	2.04	1.81	1.65	1.54	1.45	1.39	1.33	1.29	1.25	1.22	1.19	1.17	1.15	1.13	1.11	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.03	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	
11	2.39	2.09	1.76	1.51	1.30	1.21	1.13	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
12	2.33	2.04	1.86	1.72	1.57	1.46	1.38	1.27	1.20	1.15	1.12	1.10	1.08	1.06	1.04	1.03	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
13	2.29	2.01	1.81	1.69	1.54	1.43	1.35	1.29	1.23	1.19	1.15	1.12	1.10	1.07	1.07	1.05	1.03	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
14	2.25	1.87	1.66	1.51	1.40	1.32	1.26	1.20	1.17	1.13	1.10	1.07	1.05	1.03	1.01	0.99	0.98	0.97	0.96	0.95	0.94	0.94	0.92	0.92	0.91	0.91	0.90	0.90	0.90	0.90	0.90	0.90	0.90		
15	2.22	1.85	1.63	1.49	1.39	1.30	1.26	1.20	1.15	1.13	1.10	1.08	1.05	1.03	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.86	0.85	0.85	0.85	0.85	0.85		
16	2.19	1.82	1.61	1.47	1.36	1.28	1.22	1.17	1.13	1.09	1.05	1.03	1.01	0.99	0.97	0.96	0.94	0.93	0.92	0.90	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.84	0.84	0.84	0.84	0.84	0.84		
17	2.17	1.80	1.65	1.45	1.34	1.27	1.20	1.15	1.11	1.06	1.02	1.00	0.97	0.95	0.93	0.92	0.90	0.89	0.88	0.87	0.86	0.85	0.85	0.84	0.83	0.83	0.82	0.82	0.82	0.82	0.82	0.82	0.82		
18	2.15	1.78	1.57	1.43	1.33	1.25	1.19	1.14	1.10	1.06	1.03	1.00	0.98	0.96	0.94	0.92	0.90	0.89	0.88	0.87	0.86	0.84	0.83	0.82	0.82	0.82	0.81	0.81	0.81	0.81	0.81	0.81	0.81		
19	2.13	1.77	1.56	1.42	1.31	1.24	1.17	1.12	1.08	1.05	1.02	1.00	0.99	0.97	0.95	0.93	0.92	0.90	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.82	0.81	0.80	0.80	0.80	0.80	0.80	0.80	0.80	
20	2.11	1.75	1.54	1.40	1.30	1.20	1.15	1.11	1.07	1.03	1.00	0.98	0.96	0.94	0.92	0.90	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.77	0.77	0.77	0.77	
21	2.09	1.76	1.53	1.39	1.29	1.20	1.14	1.09	1.05	1.02	1.00	0.97	0.95	0.93	0.90	0.89	0.87	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	
22	2.08	1.75	1.52	1.38	1.28	1.20	1.14	1.09	1.05	1.02	1.00	0.98	0.96	0.94	0.92	0.90	0.89	0.87	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	
23	2.07	1.72	1.51	1.37	1.27	1.20	1.13	1.08	1.04	1.01	0.98	0.95	0.92	0.90	0.89	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	
24	2.06	1.71	1.50	1.36	1.26	1.18	1.12	1.07	1.04	1.00	0.97	0.94	0.92	0.90	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	
25	2.05	1.70	1.49	1.35	1.25	1.18	1.12	1.07	1.03	0.99	0.96	0.93	0.91	0.89	0.87	0.85	0.84	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	
26	2.04	1.69	1.48	1.35	1.25	1.17	1.11	1.06	1.02	0.98	0.95	0.93	0.90	0.88	0.86	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	
27	2.03	1.68	1.48	1.34	1.24	1.16	1.09	1.04	1.00	0.97	0.95	0.92	0.90	0.87	0.85	0.83	0.81	0.79	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61
28	2.02	1.67	1.47	1.34	1.24	1.16	1.09	1.04	1.00	0.97	0.95	0.92	0.90	0.87	0.85	0.83	0.81	0.79	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61
29	2.01	1.67	1.47	1.33	1.23	1.16	1.09	1.04	1.00	0.97	0.95	0.92	0.90	0.87	0.85	0.83	0.81	0.79	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61
30	2.00	1.67	1.46	1.33	1.23	1.15	1.08	1.03	0.99	0.96	0.93	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61
31	1.99	1.59	1.38	1.26	1.16	1.08	1.02	0.98	0.94	0.91	0.88	0.85	0.82	0.80	0.78	0.76	0.74	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55
32	1.98	1.58	1.37	1.25	1.15	1.07	1.01	0.97	0.93	0.90	0.87	0.84	0.81	0.79	0.77	0.75	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55
33	1.97	1.57	1.37	1.24	1.14	1.06	1.00	0.96	0.92	0.89	0.86	0.83	0.80	0.78	0.76	0.74	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	
34	1.96	1.56	1.36	1.23	1.13	1.05	0.99	0.95	0.91	0.88	0.85	0.82	0.79	0.76	0.74	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55		
35	1.95	1.55	1.35	1.22	1.12	1.04	0.98	0.94	0.90	0.87	0.84	0.81	0.78	0.75	0.73	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55			
36	1.95	1.54	1.34	1.21	1.11	1.03	0.97	0.93	0.89	0.86	0.83	0.80	0.77	0.74	0.72	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55				
37	1.94	1.53	1.33	1.20	1.09	1.01	0.95	0.91	0.87	0.84	0.81	0.78	0.75	0.72	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55					
38	1.93	1.52	1.32	1.19	1.08	1.00																													

$$\text{Value of D} = \delta/S_D$$

- A B L E 6

Single-sided test  $\alpha = .01$        $\beta = .01$   
 Double-sided test  $\alpha = .02$

$$\text{Value of } D = \delta/S_p$$

TABLE 7  
Single-sided test  $\alpha = .01$      $\beta = .05$   
Double-sided test  $\alpha = .02$

n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	35	40	45	50	60	70	80				
2	9.88	6.29	5.09	4.50	4.15	3.92	3.76	3.64	3.54	3.47	3.41	3.36	3.31	3.23	3.25	3.22	3.19	3.17	3.15	3.14	3.12	3.11	3.09	3.08	3.07	3.06	3.05	3.04	3.03	3.00	2.98	2.96	2.94	2.92	2.90	2.89				
3	6.29	4.80	4.11	3.71	3.46	3.28	3.15	3.04	2.97	2.91	2.85	2.81	2.77	2.74	2.71	2.68	2.66	2.64	2.62	2.61	2.59	2.58	2.55	2.54	2.52	2.50	2.48	2.46	2.44	2.42	2.40	2.39	2.38							
4	5.09	4.11	3.60	3.28	3.07	2.92	2.56	2.71	2.64	2.58	2.53	2.49	2.45	2.42	2.40	2.37	2.35	2.33	2.31	2.30	2.28	2.27	2.26	2.25	2.24	2.23	2.22	2.21	2.18	2.15	2.13	2.12	2.11	2.08	2.07					
5	4.50	3.71	3.28	3.01	2.82	2.68	2.57	2.49	2.42	2.37	2.32	2.26	2.24	2.21	2.19	2.16	2.14	2.12	2.11	2.09	2.08	2.07	2.05	2.03	2.02	2.01	1.97	1.95	1.93	1.91	1.89	1.87	1.86							
6	4.15	3.46	3.07	2.82	2.64	2.51	2.41	2.33	2.26	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.99	1.97	1.96	.94	1.93	1.91	1.89	1.88	1.87	1.86	1.86	1.85	1.84	1.83	1.82	1.81	1.80	1.78	1.77	1.76	1.74	1.72	1.71	
7	3.92	3.28	2.92	2.68	2.51	2.39	2.29	2.21	2.15	2.09	2.05	2.01	1.98	1.95	1.92	1.90	1.88	1.86	1.84	1.83	1.81	1.79	1.77	1.76	1.75	1.73	1.70	1.68	1.66	1.64	1.62	1.60	1.59	1.58						
8	3.76	3.15	2.80	2.57	2.41	2.29	2.19	2.12	2.05	2.00	1.96	1.92	1.88	1.86	1.83	1.81	1.79	1.77	1.75	1.74	1.72	1.71	1.70	1.69	1.68	1.67	1.66	1.65	1.64	1.61	1.58	1.57	1.55	1.53	1.51	1.50				
9	3.64	3.05	2.71	2.49	2.33	2.21	2.12	2.04	1.98	1.93	1.88	1.84	1.81	1.78	1.76	1.73	1.71	1.69	1.68	1.66	1.65	1.63	1.62	1.61	1.60	1.58	1.57	1.53	1.51	1.49	1.47	1.45	1.43	1.42	1.41	1.40				
10	3.54	2.97	2.64	2.42	2.26	2.15	2.05	1.98	1.92	1.86	1.82	1.78	1.75	1.72	1.69	1.67	1.65	1.63	1.61	1.60	1.58	1.57	1.56	1.55	1.53	1.52	1.51	1.51	1.47	1.45	1.43	1.41	1.39	1.37	1.35					
11	3.47	2.91	2.58	2.37	2.21	2.09	2.09	2.11	1.93	1.86	1.81	1.77	1.73	1.70	1.67	1.64	1.62	1.60	1.58	1.56	1.55	1.53	1.52	1.51	1.50	1.48	1.47	1.46	1.44	1.42	1.39	1.37	1.36	1.33	1.31	1.30				
12	3.41	2.85	2.53	2.32	2.16	2.05	1.96	1.88	1.82	1.77	1.72	1.69	1.65	1.62	1.60	1.57	1.55	1.53	1.52	1.50	1.49	1.47	1.45	1.43	1.42	1.40	1.37	1.34	1.32	1.31	1.28	1.26	1.25							
13	3.36	2.81	2.49	2.28	2.13	2.01	1.92	1.84	1.78	1.73	1.69	1.64	1.61	1.56	1.54	1.51	1.50	1.48	1.47	1.45	1.42	1.41	1.40	1.39	1.37	1.36	1.33	1.30	1.28	1.26	1.24	1.22	1.21	1.20	1.20					
14	3.31	2.77	2.45	2.24	2.09	1.98	1.88	1.81	1.75	1.70	1.65	1.61	1.58	1.55	1.53	1.50	1.48	1.46	1.45	1.42	1.41	1.40	1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.27	1.25	1.23	1.20	1.18	1.17			
15	3.25	2.74	2.42	2.21	2.06	1.95	1.86	1.78	1.72	1.67	1.62	1.59	1.55	1.52	1.50	1.47	1.45	1.43	1.42	1.40	1.38	1.37	1.35	1.34	1.32	1.31	1.30	1.26	1.23	1.21	1.20	1.17	1.15	1.13						
16	3.25	2.71	2.40	2.19	2.04	1.92	1.83	1.76	1.69	1.64	1.60	1.56	1.53	1.50	1.47	1.45	1.43	1.41	1.39	1.37	1.36	1.34	1.32	1.30	1.29	1.28	1.27	1.23	1.21	1.18	1.17	1.16	1.14	1.12	1.10					
17	3.22	2.68	2.37	2.16	2.01	1.90	1.81	1.73	1.67	1.62	1.57	1.54	1.50	1.47	1.45	1.42	1.40	1.39	1.36	1.35	1.33	1.32	1.31	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.22	1.21	1.18	1.16	1.14	1.11	1.09	1.08		
18	3.19	2.66	2.35	2.14	2.09	1.98	1.88	1.81	1.75	1.70	1.65	1.60	1.55	1.51	1.49	1.45	1.43	1.40	1.38	1.36	1.34	1.31	1.30	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.18	1.17	1.16	1.14	1.13	1.12	1.10			
19	3.17	2.64	2.33	2.12	1.97	1.86	1.77	1.69	1.63	1.58	1.53	1.50	1.46	1.43	1.41	1.39	1.36	1.34	1.32	1.31	1.29	1.27	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.18	1.17	1.16	1.15	1.14	1.13	1.12				
20	3.15	2.62	2.31	2.11	1.96	1.84	1.75	1.68	1.61	1.56	1.52	1.48	1.45	1.42	1.39	1.36	1.33	1.31	1.28	1.26	1.24	1.23	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.07	1.05	1.02	1.01
21	3.14	2.61	2.30	2.09	1.94	1.83	1.74	1.66	1.60	1.55	1.50	1.47	1.43	1.40	1.37	1.35	1.33	1.31	1.29	1.27	1.25	1.24	1.23	1.22	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.07	1.06	1.03	1.00	99
22	3.12	2.59	2.20	2.08	1.93	1.81	1.72	1.65	1.58	1.53	1.49	1.45	1.42	1.38	1.36	1.33	1.31	1.29	1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.05	97	
23	3.11	2.58	2.27	2.07	1.91	1.80	1.71	1.63	1.57	1.52	1.48	1.44	1.40	1.37	1.34	1.32	1.30	1.27	1.25	1.23	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	95			
24	3.09	2.57	2.26	2.05	1.90	1.79	1.70	1.62	1.56	1.51	1.47	1.42	1.39	1.36	1.33	1.31	1.28	1.26	1.24	1.22	1.20	1.18	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	93				
25	3.08	2.56	2.25	2.04	1.89	1.76	1.69	1.61	1.55	1.50	1.45	1.41	1.36	1.35	1.32	1.29	1.27	1.25	1.23	1.22	1.20	1.18	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	91					
26	3.07	2.55	2.24	2.03	1.88	1.77	1.68	1.61	1.55	1.48	1.44	1.40	1.37	1.34	1.30	1.28	1.26	1.24	1.22	1.20	1.19	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	90				
27	3.06	2.54	2.23	2.02	1.87	1.76	1.67	1.60	1.53	1.48	1.43	1.39	1.36	1.32	1.30	1.27	1.25	1.23	1.21	1.19	1.18	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	90					
28	3.05	2.53	2.22	2.02	1.87	1.76	1.67	1.58	1.52	1.47	1.42	1.39	1.34	1.31	1.29	1.26	1.24	1.22	1.20	1.18	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	90						
29	3.04	2.52	2.22	2.01	1.86	1.75	1.65	1.58	1.51	1.46	1.42	1.37	1.34	1.31	1.28	1.25	1.23	1.21	1.19	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	90						
30	3.03	2.52	2.21	2.01	1.86	1.73	1.64	1.57	1.51	1.46	1.40	1.36	1.33	1.30	1.27	1.24	1.22	1.20	1.18	1.16	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	90								
35	3.00	2.48	2.18	2.08	1.82	1.70	1.61	1.5																																

TABLE 8  
Single-sided test  $\alpha = .01$        $\beta = .$

Value of D =  $\delta/S_p$

TABLE 9

Single-sided test $\alpha = .01$	$B = .20$
Double-sided test $\alpha = .02$	

$n_1$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	35	40	45	50	60	70	80				
2	8.93	5.04	4.06	3.59	3.31	3.12	2.99	2.90	2.86	2.76	2.71	2.67	2.64	2.61	2.59	2.56	2.53	2.51	2.50	2.49	2.48	2.46	2.44	2.43	2.42	2.42	2.39	2.37	2.33	2.31	2.31	2.31								
3	5.04	3.83	3.27	2.96	2.75	2.61	2.51	2.43	2.37	2.31	2.27	2.24	2.21	2.18	2.16	2.13	2.12	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.01	2.01	2.00	1.96	1.95	1.93	1.92	1.90	1.89					
4	4.06	3.27	2.86	2.61	2.44	2.32	2.23	2.16	2.10	2.06	2.02	1.96	1.95	1.93	1.91	1.89	1.87	1.86	1.84	1.83	1.82	1.80	1.79	1.78	1.76	1.74	1.72	1.70	1.69	1.67	1.66	1.65	1.64	1.63						
5	3.59	2.96	2.61	2.39	2.24	2.13	2.05	1.98	1.93	1.88	1.85	1.81	1.79	1.74	1.71	1.69	1.68	1.67	1.66	1.65	1.64	1.63	1.62	1.61	1.61	1.60	1.60	1.57	1.55	1.53	1.51	1.49	1.48							
6	3.31	2.75	2.44	2.24	2.10	2.00	1.88	1.86	1.80	1.76	1.72	1.69	1.67	1.64	1.62	1.60	1.59	1.57	1.56	1.55	1.54	1.53	1.52	1.51	1.50	1.49	1.48	1.45	1.43	1.42	1.41	1.39	1.37	1.36	1.35					
7	3.12	2.61	2.32	2.13	2.00	1.90	1.82	1.76	1.71	1.67	1.63	1.60	1.57	1.55	1.53	1.51	1.50	1.48	1.47	1.46	1.44	1.43	1.42	1.41	1.40	1.40	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27					
8	2.99	2.51	2.23	2.05	1.92	1.82	1.75	1.69	1.64	1.59	1.56	1.53	1.50	1.48	1.46	1.44	1.42	1.41	1.40	1.39	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.22	1.20	1.19				
9	2.90	2.43	2.16	1.98	1.86	1.76	1.69	1.63	1.59	1.53	1.50	1.47	1.44	1.42	1.40	1.38	1.36	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.19	1.17	1.16	1.14	1.13			
10	2.82	2.37	2.10	1.93	1.80	1.71	1.64	1.58	1.53	1.49	1.45	1.42	1.39	1.37	1.35	1.33	1.32	1.30	1.29	1.27	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.17	1.15	1.14	1.12	1.11	1.09	1.08	1.06	1.05	1.05			
11	2.76	2.31	2.06	1.88	1.76	1.67	1.59	1.53	1.49	1.44	1.41	1.38	1.35	1.33	1.31	1.29	1.27	1.26	1.24	1.23	1.22	1.21	1.20	1.18	1.18	1.17	1.17	1.16	1.13	1.11	1.09	1.08	1.06	1.05	1.05					
12	2.71	2.27	2.02	1.85	1.72	1.63	1.56	1.50	1.45	1.41	1.37	1.34	1.32	1.29	1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.06	1.05	1.05					
13	2.67	2.24	1.98	1.81	1.69	1.60	1.53	1.47	1.42	1.38	1.34	1.31	1.29	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.06	1.04	1.02	1.01	1.01	1.00	1.00					
14	2.64	2.21	1.95	1.79	1.67	1.57	1.50	1.44	1.40	1.35	1.32	1.29	1.26	1.24	1.22	1.20	1.18	1.17	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.07	1.06	1.03	1.01	1.00	1.00	1.00	1.00					
15	2.61	2.18	1.93	1.76	1.64	1.55	1.48	1.42	1.37	1.33	1.29	1.26	1.24	1.21	1.19	1.17	1.16	1.14	1.13	1.12	1.10	1.09	1.08	1.07	1.07	1.05	1.05	1.04	1.04	1.03	1.03	1.03	1.03	1.03	1.03					
16	2.59	2.16	1.91	1.74	1.62	1.53	1.46	1.40	1.35	1.31	1.27	1.24	1.22	1.19	1.17	1.15	1.14	1.12	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03				
17	2.56	2.14	1.89	1.72	1.60	1.51	1.44	1.38	1.33	1.29	1.25	1.22	1.20	1.17	1.15	1.14	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00				
18	2.54	2.12	1.87	1.71	1.59	1.50	1.42	1.36	1.32	1.27	1.24	1.21	1.18	1.16	1.14	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00					
19	2.53	2.10	1.86	1.69	1.57	1.48	1.41	1.35	1.30	1.26	1.22	1.19	1.17	1.14	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00						
20	2.51	2.09	1.84	1.68	1.56	1.47	1.40	1.34	1.29	1.24	1.21	1.18	1.15	1.13	1.11	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00					
21	2.50	2.08	1.83	1.67	1.55	1.46	1.38	1.32	1.27	1.23	1.20	1.17	1.14	1.12	1.10	1.07	1.06	1.04	1.03	1.02	1.01	1.00	98	97	96	95	94	93	92	91	90	89	88	87	86					
22	2.49	2.07	1.82	1.66	1.54	1.44	1.37	1.31	1.26	1.22	1.19	1.16	1.13	1.10	1.08	1.06	1.05	1.04	1.03	1.02	1.01	1.00	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84		
23	2.48	2.06	1.81	1.65	1.53	1.43	1.36	1.30	1.25	1.21	1.18	1.15	1.12	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83		
24	2.46	2.05	1.80	1.64	1.52	1.42	1.35	1.29	1.25	1.21	1.17	1.14	1.10	1.07	1.05	1.03	1.01	99	98	97	96	95	94	93	92	90	89	88	87	86	85	84	83	82	80	78	76	75	74	
25	2.46	2.04	1.79	1.63	1.51	1.42	1.34	1.29	1.24	1.20	1.16	1.12	1.09	1.07	1.04	1.02	1.00	98	96	95	93	92	91	90	89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74
26	2.42	2.03	1.78	1.62	1.50	1.41	1.34	1.28	1.23	1.18	1.15	1.12	1.09	1.06	1.04	1.02	1.00	97	96	94	93	91	90	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74		
27	2.41	2.02	1.76	1.61	1.49	1.40	1.33	1.27	1.22	1.18	1.14	1.11	1.08	1.05	1.03	1.01	98	96	95	93	91	90	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74			
28	2.43	2.01	1.77	1.61	1.49	1.40	1.33	1.26	1.21	1.17	1.14	1.10	1.07	1.04	1.01	99	98	96	95	93	92	90	89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74		
29	2.42	2.01	1.77	1.60	1.48	1.39	1.31	1.26	1.21	1.17	1.13	1.09	1.07	1.05	1.03	1.01	98	96	95	93	92	91	90	89	88	87	86	85	84	83	82	81	80	79	78	77	76	75		
30	2.42	2.00	1.76	1.60	1.46	1.38	1.31	1.25	1.20	1.16	1.12	1.09	1.06	1.04	1.02	1.00	97	96	95	93	91	90	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74			
31	2.39	1.98	1.78	1.57	1.45	1.36	1.28	1.22	1.17	1.13	1.09	1.06	1.03	1.01	98	96	95	93	91	90	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74					
32	2.37	2.01	1.76																																					

TABLE 10  
Single-sided test  $\alpha = .01$

Country		GDP		Population		Emissions		Emissions Intensity		Emissions per Capita		Emissions per GDP		Emissions per Capita per GDP	
Code	Name	Value	Unit	Value	Unit	Value	Unit	Value	Unit	Value	Unit	Value	Unit	Value	Unit
6.96	4.15	3.24	\$B	2.02	2.57	2.40	\$B	2.29	2.21	2.14	\$B	2.05	2.01	1.98	\$B
3.15	2.06	2.57	\$B	2.30	2.12	2.00	\$B	1.91	1.04	1.79	\$B	1.96	1.92	1.82	\$B
4.21	2.57	2.22	\$B	2.01	1.87	1.77	\$B	1.69	1.63	1.59	\$B	1.46	1.44	1.43	\$B
5.25	2.30	2.01	\$B	1.83	1.71	1.62	\$B	1.55	1.51	1.49	\$B	1.34	1.32	1.30	\$B
6.25	2.12	1.87	\$B	1.71	1.62	1.55	\$B	1.45	1.42	1.39	\$B	1.25	1.23	1.21	\$B
7.20	2.00	1.77	\$B	1.62	1.57	1.51	\$B	1.50	1.46	1.42	\$B	1.27	1.25	1.23	\$B
8.	2.29	1.91	\$B	1.69	1.55	1.45	\$B	1.37	1.31	1.26	\$B	1.12	1.10	1.09	\$B
9.	2.21	1.84	\$B	1.63	1.50	1.40	\$B	1.32	1.26	1.22	\$B	1.15	1.13	1.11	\$B
10.	2.14	1.79	\$B	1.59	1.45	1.36	\$B	1.28	1.23	1.19	\$B	1.06	1.04	1.03	\$B
11.	2.09	1.75	\$B	1.55	1.42	1.32	\$B	1.25	1.25	1.22	\$B	1.05	1.03	1.01	\$B
12.	2.05	1.71	\$B	1.51	1.39	1.29	\$B	1.22	1.16	1.12	\$B	1.01	1.00	1.00	\$B
13.	2.01	1.68	\$B	1.49	1.36	1.27	\$B	1.20	1.14	1.10	\$B	0.93	0.90	0.89	\$B
14.	1.90	1.66	\$B	1.46	1.34	1.25	\$B	1.18	1.12	1.08	\$B	0.91	0.89	0.87	\$B
15.	1.96	1.63	\$B	1.44	1.32	1.23	\$B	1.16	1.10	1.06	\$B	0.90	0.86	0.83	\$B
16.	1.94	1.62	\$B	1.43	1.30	1.21	\$B	1.14	1.09	1.04	\$B	0.95	0.90	0.86	\$B
17.	1.92	1.60	\$B	1.41	1.29	1.20	\$B	1.13	1.07	1.03	\$B	0.96	0.91	0.86	\$B
18.	1.90	1.58	\$B	1.40	1.27	1.19	\$B	1.11	1.06	1.01	\$B	0.95	0.90	0.86	\$B
19.	1.89	1.57	\$B	1.39	1.26	1.21	\$B	1.10	1.05	1.00	\$B	0.93	0.91	0.88	\$B
20.	1.87	1.56	\$B	1.37	1.25	1.16	\$B	1.09	1.04	0.99	\$B	0.90	0.86	0.82	\$B
21.	1.86	1.55	\$B	1.36	1.24	1.15	\$B	1.08	1.03	0.98	\$B	0.89	0.87	0.85	\$B
22.	1.85	1.54	\$B	1.35	1.23	1.14	\$B	1.07	1.02	0.97	\$B	0.91	0.86	0.82	\$B
23.	1.84	1.53	\$B	1.35	1.22	1.13	\$B	1.06	1.01	0.97	\$B	0.90	0.87	0.83	\$B
24.	1.83	1.52	\$B	1.34	1.22	1.12	\$B	1.06	1.00	0.96	\$B	0.89	0.85	0.81	\$B
25.	1.83	1.51	\$B	1.33	1.21	1.12	\$B	1.05	1.00	0.95	\$B	0.88	0.84	0.80	\$B
26.	1.84	1.51	\$B	1.32	1.20	1.11	\$B	1.05	1.00	0.95	\$B	0.86	0.82	0.78	\$B
27.	1.83	1.50	\$B	1.32	1.20	1.11	\$B	1.04	0.99	0.95	\$B	0.87	0.83	0.79	\$B
28.	1.81	1.49	\$B	1.31	1.19	1.11	\$B	1.04	0.99	0.95	\$B	0.86	0.82	0.78	\$B
29.	1.80	1.49	\$B	1.31	1.19	1.10	\$B	1.03	0.97	0.93	\$B	0.86	0.82	0.78	\$B
30.	1.79	1.49	\$B	1.31	1.19	1.16	\$B	1.02	0.97	0.93	\$B	0.86	0.82	0.78	\$B
31.	1.78	1.47	\$B	1.29	1.17	1.08	\$B	1.00	0.95	0.91	\$B	0.87	0.83	0.79	\$B
32.	1.76	1.45	\$B	1.27	1.15	1.06	\$B	1.00	0.95	0.91	\$B	0.86	0.82	0.78	\$B
33.	1.74	1.44	\$B	1.26	1.14	1.05	\$B	1.00	0.95	0.91	\$B	0.86	0.82	0.78	\$B
34.	1.73	1.43	\$B	1.25	1.13	1.04	\$B	1.00	0.95	0.91	\$B	0.87	0.83	0.79	\$B
35.	1.72	1.41	\$B	1.23	1.11	1.02	\$B	1.00	0.95	0.91	\$B	0.86	0.82	0.78	\$B
36.	1.71	1.40	\$B	1.22	1.10	1.01	\$B	1.00	0.95	0.91	\$B	0.86	0.82	0.78	\$B
37.	1.70	1.39	\$B	1.22	1.10	1.02	\$B	1.00	0.95	0.91	\$B	0.86	0.82	0.78	\$B

Value of D =  $\delta/S_p$

TABLE 11

Single-sided test  $\alpha = .025$        $\epsilon = .01$   
 Double-sided test  $\alpha = .05$

$$\text{Value of D} = \epsilon / S_p$$

TABLE 12

TABLE 13

Value of D =  $\xi/S_p$

TABLE 14

Single-sided test $\alpha = 0.05$		Double-sided test $\alpha = 0.05$	
$n_1$	$n_2$	2	3
2	5.26	3.80	3.22
3	3.80	2.92	2.76
4	3.07	2.67	2.45
5	2.82	2.45	2.19
6	2.74	2.32	2.06
7	2.63	2.20	1.97
8	2.55	2.13	1.90
9	2.46	2.07	1.85
10	2.41	2.03	1.81
11	2.37	1.99	1.77
12	2.33	1.96	1.76
13	2.30	1.93	1.73
14	2.28	1.91	1.69
15	2.26	1.89	1.67
16	2.24	1.87	1.66
17	2.22	1.86	1.64
18	2.21	1.83	1.63
19	2.20	1.83	1.62
20	2.18	1.82	1.61
21	2.17	1.81	1.60
22	2.15	1.79	1.57
23	2.14	1.78	1.57
24	2.13	1.76	1.57
25	2.12	1.75	1.56
26	2.11	1.75	1.55
27	2.11	1.75	1.55
28	2.10	1.73	1.55
29	2.11	1.73	1.55
30	2.08	1.67	1.45
31	2.05	1.59	1.37
32	2.02	1.52	1.31
33	2.00	1.49	1.29
34	1.98	1.46	1.27
35	2.10	1.72	1.52
36	2.05	1.68	1.48
37	2.02	1.65	1.45
38	2.00	1.63	1.43
39	1.98	1.61	1.41
40	2.08	1.67	1.47
41	2.05	1.64	1.44
42	2.02	1.62	1.42
43	2.00	1.60	1.40
44	1.98	1.58	1.38
45	2.07	1.71	1.51
46	2.05	1.69	1.49
47	2.02	1.66	1.46
48	2.00	1.64	1.44
49	1.98	1.62	1.42
50	2.05	1.70	1.50

Value of D =  $\delta/S_p$

**NOT REPRODUCIBLE**

TABLE 15

$$\text{Value of D} = \epsilon / S_p$$

Value of D = 6/S<sub>p</sub>

TABLE 16

		Single-sided test, $\alpha = .05$										Double-sided test, $\alpha = .10$																																																																																
$n_1$	$n_2$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57																																			
2	9.00	6.29	5.09	4.57	4.15	3.80	3.56	3.34	3.15	2.97	2.82	2.68	2.55	2.43	2.32	2.22	2.13	2.05	1.97	1.90	1.83	1.77	1.71	1.65	1.60	1.55	1.50	1.45	1.40	1.35	1.30	1.25	1.20	1.15	1.10	1.05	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10	0.05	0.00																																			
3	6.29	4.80	4.11	3.71	3.46	3.20	3.03	2.87	2.71	2.59	2.47	2.36	2.25	2.15	2.05	1.95	1.87	1.79	1.72	1.65	1.59	1.52	1.45	1.39	1.32	1.26	1.20	1.14	1.08	1.02	0.96	0.90	0.84	0.78	0.72	0.66	0.60	0.54	0.48	0.42	0.36	0.30	0.24	0.18	0.12	0.06	0.00																																													
4	5.29	4.11	3.40	3.07	3.07	2.97	2.90	2.87	2.84	2.80	2.75	2.70	2.65	2.60	2.55	2.50	2.45	2.40	2.35	2.30	2.25	2.20	2.15	2.10	2.05	2.00	1.95	1.90	1.85	1.80	1.75	1.70	1.65	1.60	1.55	1.50	1.45	1.40	1.35	1.30	1.25	1.20	1.15	1.10	1.05	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10	0.05	0.00																										
5	4.29	3.71	3.20	2.92	2.92	2.82	2.80	2.77	2.75	2.72	2.68	2.65	2.62	2.58	2.55	2.52	2.48	2.45	2.42	2.37	2.35	2.32	2.28	2.25	2.22	2.18	2.15	2.12	2.08	2.05	2.02	1.98	1.95	1.92	1.88	1.85	1.82	1.78	1.75	1.72	1.68	1.65	1.62	1.58	1.55	1.52	1.48	1.45	1.42	1.38	1.35	1.32	1.28	1.25	1.22	1.18	1.15	1.12	1.08	1.05	1.02	0.98	0.95	0.92	0.88	0.85	0.82	0.78	0.75	0.72	0.68	0.65	0.62	0.58	0.55	0.52	0.48	0.45	0.42	0.38	0.35	0.32	0.28	0.25	0.22	0.18	0.15	0.12	0.08	0.05	0.02	0.00
6	4.19	3.46	3.07	2.82	2.82	2.64	2.51	2.43	2.32	2.25	2.18	2.12	2.05	1.98	1.92	1.86	1.80	1.74	1.68	1.62	1.56	1.50	1.44	1.38	1.32	1.26	1.20	1.14	1.08	1.02	0.96	0.90	0.84	0.78	0.72	0.66	0.60	0.54	0.48	0.42	0.36	0.30	0.24	0.18	0.12	0.06	0.00																																													
7	3.98	3.28	2.89	2.66	2.51	2.59	2.49	2.42	2.37	2.30	2.25	2.19	2.13	2.07	2.01	1.95	1.89	1.83	1.77	1.71	1.65	1.59	1.53	1.47	1.41	1.35	1.29	1.23	1.17	1.11	1.05	0.99	0.93	0.87	0.81	0.75	0.69	0.63	0.57	0.51	0.45	0.39	0.33	0.27	0.21	0.15	0.09	0.03	0.00																																											
8	3.76	3.15	2.60	2.57	2.41	2.39	2.39	2.32	2.25	2.20	2.15	2.10	2.05	2.00	1.95	1.90	1.85	1.80	1.75	1.70	1.65	1.60	1.55	1.50	1.45	1.40	1.35	1.30	1.25	1.20	1.15	1.10	1.05	1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10	0.05	0.00																																						
9	3.65	2.71	2.69	2.69	2.33	2.21	2.12	2.02	1.92	1.82	1.72	1.62	1.52	1.42	1.32	1.22	1.12	1.02	0.92	0.82	0.72	0.62	0.52	0.42	0.32	0.22	0.12	0.02	0.00																																																															
10	3.54	2.67	2.42	2.36	2.15	2.05	1.98	1.90	1.82	1.75	1.67	1.59	1.51	1.43	1.35	1.27	1.19	1.11	1.03	0.95	0.87	0.79	0.71	0.63	0.55	0.47	0.39	0.31	0.23	0.15	0.07	0.00																																																												
11	3.44	2.59	2.38	2.27	2.11	2.01	1.92	1.82	1.72	1.62	1.52	1.42	1.32	1.22	1.12	1.02	0.92	0.82	0.72	0.62	0.52	0.42	0.32	0.22	0.12	0.02	0.00																																																																	
12	3.43	2.55	2.33	2.22	2.11	2.01	1.92	1.82	1.72	1.62	1.52	1.42	1.32	1.22	1.12	1.02	0.92	0.82	0.72	0.62	0.52	0.42	0.32	0.22	0.12	0.02	0.00																																																																	
13	3.38	2.51	2.28	2.13	2.01	1.92	1.82	1.72	1.62	1.52	1.42	1.32	1.22	1.12	1.02	0.92	0.82	0.72	0.62	0.52	0.42	0.32	0.22	0.12	0.02	0.00																																																																		
14	3.31	2.45	2.26	2.09	1.98	1.88	1.78	1.68	1.58	1.48	1.38	1.28	1.18	1.08	0.98	0.88	0.78	0.68	0.58	0.48	0.38	0.28	0.18	0.08	0.00																																																																			
15	3.20	2.36	2.12	2.01	1.91	1.81	1.71	1.61	1.51	1.41	1.31	1.21	1.11	1.01	0.91	0.81	0.71	0.61	0.51	0.41	0.31	0.21	0.11	0.01	0.00																																																																			
16	3.17	2.33	2.10	2.00	1.89	1.79	1.69	1.59	1.49	1.39	1.29	1.19	1.09	0.99	0.89	0.79	0.69	0.59	0.49	0.39	0.29	0.19	0.09	0.00																																																																				
17	3.02	2.26	2.04	1.94	1.84	1.74	1.64	1.54	1.44	1.34	1.24	1.14	1.04	0.94	0.84	0.74	0.64	0.54	0.44	0.34	0.24	0.14	0.04	0.00																																																																				
18	2.95	2.19	1.96	1.86	1.76	1.66	1.56	1.46	1.36	1.26	1.16	1.06	0.96	0.86	0.76	0.66	0.56	0.46	0.36	0.26	0.16	0.06	0.00																																																																					
19	2.87	2.11	1.88	1.78	1.68	1.58	1.48	1.38	1.28	1.18	1.08	0.98	0.88	0.78	0.68	0.58	0.48	0.38	0.28	0.18	0.08	0.00																																																																						
20	2.85	2.08	1.85	1.75	1.65	1.55	1.45	1.35	1.25	1.15	1.05	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05	0.00																																																																						
21	2.84	2.06	1.84	1.74	1.64	1.54	1.44	1.34	1.24	1.14	1.04	0.94	0.84	0.74	0.64	0.54	0.44	0.34	0.24	0.14	0.04	0.00																																																																						
22	2.82	2.04	1.82	1.72	1.62	1.52	1.42	1.32	1.22	1.12	1.02	0.92	0.82	0.72	0.62	0.52	0.42	0.32	0.22	0.12	0.02	0.00																																																																						
23	2.81	2.03	1.81	1.71	1.61	1.51	1.41	1.31	1.21	1.11	1.01	0.91	0.81	0.71	0.61	0.51	0.41	0.31	0.21	0.11	0.01	0.00																																																																						
24	2.79	2.02	1.80	1.69	1.59	1.49	1.39	1.29	1.19	1.09	0.99	0.89	0.79	0.69	0.59	0.49	0.39	0.29	0.19	0.09	0.00																																																																							
25	2.78	2.01	1.79	1.68	1.58	1.48	1.38	1.28	1.18	1.08	0.98	0.88	0.78	0.68	0.58	0.48	0.38	0.28	0.18	0.08	0.00																																																																							
26	2.77	2.00	1.78	1.67	1.57	1.47	1.37	1.27	1.17	1.07	0.97	0.87	0.77	0.67	0.57	0.47	0.37	0.27	0.17	0.07	0.00																																																																							
27	2.75	1.99	1.77	1.66	1.56	1.46	1.36	1.26	1.16	1.06	0.96	0.86	0.76	0.66	0.56	0.46	0.36	0.26	0.16	0.06	0.00																																																																							
28	2.74	1.98	1.76	1.65	1.55	1.45	1.35	1.25	1.15	1.05	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05	0.00																																																																							
29	2.73	1.97	1.75	1.64	1.54	1.44	1.34	1.24	1.14	1.04	0.94	0.84	0.74	0.64	0.54	0.44	0.34	0.24	0.14	0.04	0.00																																																																							
30	2.72	1.96	1.74	1.63	1.53	1.43	1.33	1.23	1.13	1.03	0.93	0.83	0.73	0.63	0.53	0.43	0.33	0.23	0.13	0.03	0.00																																																																							
31	2.70	1.94	1.73	1.62	1.52	1.42	1.32	1.22	1.12	1.02	0.92	0.82	0.72	0.62	0.52	0.42	0.32	0.22	0.12	0.02	0.00																																																																							
32	2.68	1.93	1.72	1.61	1.51	1.41	1.31	1.21</td																																																																																				

Value of D =  $\delta/S_p$

TABLE 17

n	Single-sided test $\alpha = .05$										Double-sided test $\alpha = .10$										Single-sided test $\alpha = .05$														
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	35	40	45	50	60	70
2	5.04	4.90	3.97	3.17	3.04	2.96	2.87	2.81	2.76	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.55	2.54	2.52	2.51	2.50	2.49	2.48	2.47	2.46	2.45	2.44	2.43	2.42	2.41	2.40	2.39	2.38		
3	4.30	3.48	2.84	2.48	2.37	2.48	2.42	2.36	2.32	2.29	2.26	2.23	2.21	2.19	2.17	2.15	2.14	2.13	2.12	2.11	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.01	1.99	1.98	1.97	1.96	
4	3.67	2.68	2.79	2.54	2.40	2.30	2.22	2.16	2.12	2.07	2.03	2.00	1.98	1.96	1.94	1.92	1.90	1.89	1.88	1.87	1.86	1.85	1.84	1.83	1.82	1.81	1.80	1.79	1.78	1.77	1.76	1.75	1.74	1.73	1.72
5	3.37	2.84	2.54	2.35	2.22	2.12	2.05	1.99	1.94	1.90	1.87	1.83	1.81	1.79	1.77	1.75	1.73	1.71	1.69	1.68	1.67	1.66	1.65	1.64	1.63	1.62	1.61	1.60	1.59	1.58	1.57	1.56	1.55	1.54	1.53
6	3.17	2.68	2.40	2.23	2.09	2.00	1.92	1.87	1.82	1.78	1.73	1.69	1.65	1.61	1.57	1.53	1.49	1.45	1.42	1.39	1.36	1.33	1.30	1.27	1.24	1.21	1.18	1.15	1.12	1.09	1.06	1.03	1.00	1.00	1.00
7	3.04	2.57	2.30	2.12	2.00	1.91	1.83	1.77	1.73	1.69	1.66	1.63	1.60	1.56	1.52	1.49	1.45	1.42	1.39	1.36	1.33	1.30	1.27	1.24	1.21	1.18	1.15	1.12	1.09	1.06	1.03	1.00	1.00	1.00	
8	2.96	2.48	2.22	2.05	1.92	1.83	1.76	1.70	1.66	1.62	1.58	1.55	1.52	1.49	1.46	1.43	1.40	1.37	1.34	1.31	1.29	1.26	1.23	1.20	1.17	1.14	1.11	1.08	1.05	1.02	1.00	1.00	1.00		
9	2.87	2.42	2.16	1.99	1.87	1.77	1.70	1.65	1.60	1.56	1.52	1.48	1.45	1.41	1.38	1.35	1.32	1.29	1.26	1.23	1.20	1.17	1.14	1.11	1.08	1.05	1.02	1.00	1.00	1.00	1.00	1.00	1.00		
10	2.82	2.36	2.11	1.94	1.82	1.73	1.66	1.60	1.55	1.51	1.48	1.45	1.42	1.39	1.36	1.33	1.30	1.27	1.24	1.21	1.18	1.15	1.12	1.09	1.06	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00		
11	2.76	2.32	2.07	1.90	1.79	1.69	1.62	1.56	1.51	1.46	1.41	1.36	1.32	1.28	1.25	1.22	1.19	1.16	1.13	1.10	1.07	1.04	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		
12	2.72	2.29	2.05	1.87	1.75	1.66	1.59	1.50	1.45	1.40	1.36	1.32	1.28	1.24	1.20	1.16	1.12	1.08	1.04	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
13	2.69	2.26	2.00	1.84	1.72	1.63	1.55	1.50	1.44	1.37	1.31	1.25	1.20	1.15	1.10	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		
14	2.66	2.23	1.99	1.81	1.69	1.60	1.53	1.47	1.42	1.38	1.35	1.32	1.29	1.27	1.25	1.23	1.21	1.19	1.16	1.13	1.10	1.07	1.04	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
15	2.64	2.21	1.96	1.79	1.67	1.59	1.52	1.45	1.40	1.36	1.32	1.29	1.25	1.21	1.17	1.13	1.09	1.05	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
16	2.62	2.19	1.94	1.77	1.65	1.56	1.49	1.43	1.38	1.34	1.30	1.26	1.22	1.18	1.14	1.10	1.06	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
17	2.60	2.17	1.92	1.76	1.65	1.56	1.47	1.42	1.36	1.32	1.28	1.25	1.21	1.17	1.13	1.09	1.05	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
18	2.59	2.16	1.91	1.76	1.62	1.53	1.45	1.39	1.35	1.31	1.27	1.24	1.21	1.19	1.15	1.11	1.07	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
19	2.57	2.14	1.89	1.73	1.61	1.51	1.44	1.38	1.33	1.29	1.25	1.22	1.19	1.15	1.11	1.07	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
20	2.56	2.13	1.88	1.71	1.59	1.50	1.43	1.37	1.32	1.28	1.24	1.20	1.16	1.12	1.08	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
21	2.55	2.12	1.87	1.70	1.58	1.50	1.43	1.37	1.32	1.28	1.24	1.20	1.16	1.12	1.08	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
22	2.54	2.11	1.86	1.69	1.57	1.48	1.42	1.34	1.34	1.29	1.25	1.22	1.19	1.16	1.13	1.10	1.07	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
23	2.53	2.10	1.85	1.68	1.56	1.47	1.39	1.33	1.29	1.24	1.21	1.17	1.13	1.10	1.06	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
24	2.52	2.09	1.84	1.67	1.55	1.46	1.39	1.33	1.29	1.26	1.22	1.18	1.14	1.10	1.06	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
25	2.51	2.08	1.83	1.67	1.56	1.45	1.38	1.32	1.27	1.23	1.19	1.15	1.11	1.07	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
26	2.50	2.08	1.82	1.66	1.54	1.45	1.37	1.31	1.26	1.22	1.18	1.14	1.10	1.06	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
27	2.50	2.07	1.82	1.65	1.53	1.44	1.37	1.31	1.26	1.22	1.18	1.14	1.10	1.06	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
28	2.49	2.06	1.81	1.65	1.53	1.43	1.36	1.30	1.25	1.20	1.17	1.13	1.09	1.05	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
29	2.48	2.05	1.81	1.64	1.52	1.43	1.35	1.29	1.24	1.20	1.16	1.12	1.08	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
30	2.47	2.05	1.81	1.62	1.51	1.42	1.34	1.28	1.23	1.19	1.15	1.11	1.07	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
31	2.46	2.05	1.81	1.62	1.51	1.42	1.33	1.27	1.22	1.18	1.14																								

TABLE 18

		Single-sided test $\alpha = .05$										$\beta = .10$																									
		Double-sided test $\alpha = .10$																																			
$n_1$	$n_2$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	35	40	45	50	60	70	80
2	4.61	3.64	3.37	2.82	2.76	2.65	2.57	2.51	2.47	2.43	2.40	2.37	2.35	2.33	2.31	2.30	2.28	2.27	2.26	2.25	2.24	2.23	2.22	2.21	2.20	2.20	2.20	2.20	2.16	2.15	2.13	2.12	2.12	2.12			
3	3.46	2.99	2.67	2.47	2.34	2.25	2.18	2.12	2.08	2.04	2.01	1.99	1.97	1.95	1.93	1.92	1.91	1.89	1.88	1.86	1.85	1.84	1.83	1.82	1.82	1.80	1.79	1.78	1.77	1.76	1.75	1.75	1.74				
4	3.17	2.67	2.39	2.22	2.10	2.02	1.95	1.90	1.86	1.82	1.79	1.77	1.75	1.73	1.71	1.70	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.62	1.61	1.61	1.60	1.59	1.57	1.55	1.53	1.52	1.52	1.52			
5	2.92	2.47	2.22	2.06	1.95	1.86	1.80	1.75	1.71	1.68	1.65	1.62	1.60	1.58	1.55	1.54	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45	1.45	1.43	1.42	1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.34
6	2.76	2.24	2.10	1.95	1.84	1.76	1.69	1.64	1.60	1.57	1.54	1.52	1.50	1.48	1.46	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.35	1.34	1.33	1.31	1.30	1.29	1.27	1.26	1.25		
7	2.65	2.25	2.09	1.96	1.88	1.80	1.74	1.67	1.62	1.57	1.52	1.49	1.46	1.44	1.42	1.40	1.38	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.28	1.27	1.26	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.17		
8	2.57	2.18	1.95	1.80	1.69	1.62	1.55	1.50	1.46	1.43	1.40	1.37	1.35	1.33	1.30	1.29	1.27	1.26	1.25	1.24	1.23	1.22	1.21	1.20	1.19	1.17	1.15	1.14	1.13	1.12	1.11	1.11	1.10	1.08	1.06	1.05	1.04
9	2.51	2.12	1.90	1.75	1.66	1.57	1.50	1.45	1.42	1.38	1.35	1.32	1.30	1.26	1.25	1.23	1.22	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.14	1.12	1.10	1.09	1.08	1.06	1.05	1.04	1.04			
10	2.47	2.08	1.86	1.71	1.60	1.52	1.46	1.41	1.37	1.34	1.31	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.07	1.06	1.05	1.04	1.03	1.03	1.01	1.00	0.99	
11	2.43	2.04	1.82	1.68	1.57	1.49	1.43	1.38	1.34	1.30	1.27	1.24	1.22	1.20	1.18	1.15	1.13	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.97	0.96	0.95			
12	2.40	2.00	1.79	1.65	1.56	1.46	1.40	1.35	1.31	1.27	1.24	1.21	1.19	1.17	1.15	1.14	1.12	1.11	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.03	1.02	1.01	1.00	0.98	0.96	0.95	0.92	0.91		
13	2.37	1.99	1.77	1.62	1.52	1.44	1.37	1.32	1.28	1.24	1.21	1.19	1.16	1.14	1.13	1.12	1.10	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1.01	1.00	0.99	0.97	0.95	0.94	0.92	0.91	0.89			
14	2.35	1.97	1.79	1.60	1.42	1.35	1.29	1.24	1.20	1.17	1.14	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.97	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.88	0.87	0.86		
15	2.33	1.95	1.75	1.58	1.48	1.40	1.35	1.28	1.24	1.20	1.17	1.14	1.12	1.10	1.08	1.07	1.05	1.04	1.03	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.85	
16	2.31	1.93	1.71	1.57	1.46	1.38	1.32	1.26	1.22	1.19	1.15	1.13	1.10	1.08	1.07	1.05	1.03	1.02	1.01	0.99	0.98	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.86	0.85	0.85	0.82	0.81
17	2.30	1.88	1.70	1.59	1.45	1.38	1.32	1.27	1.22	1.19	1.16	1.14	1.12	1.10	1.09	1.07	1.05	1.04	1.03	1.02	1.01	1.00	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87		
18	2.28	1.82	1.69	1.54	1.43	1.35	1.29	1.23	1.19	1.15	1.12	1.10	1.08	1.06	1.05	1.03	1.02	1.00	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.81
19	2.27	1.80	1.67	1.53	1.42	1.36	1.28	1.22	1.18	1.14	1.11	1.08	1.05	1.03	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79		
20	2.26	1.78	1.66	1.53	1.43	1.36	1.29	1.23	1.19	1.15	1.12	1.09	1.06	1.03	1.01	0.99	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79		
21	2.25	1.76	1.65	1.52	1.42	1.35	1.28	1.22	1.18	1.14	1.11	1.08	1.05	1.02	1.00	0.98	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79			
22	2.24	1.75	1.64	1.51	1.41	1.34	1.27	1.21	1.17	1.13	1.10	1.07	1.04	1.01	0.98	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77			
23	2.23	1.74	1.63	1.50	1.39	1.32	1.25	1.19	1.15	1.11	1.07	1.04	1.01	0.98	0.95	0.93	0.91	0.90	0.89	0.88	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.74			
24	2.20	1.65	1.53	1.43	1.34	1.26	1.19	1.13	1.09	1.05	1.02	0.98	0.95	0.92	0.90	0.88	0.85	0.83	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.65	
25	2.19	1.63	1.51	1.40	1.31	1.23	1.16	1.10	1.06	1.02	0.98	0.95	0.92	0.90	0.88	0.85	0.83	0.81	0.80	0.79	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64		
26	2.18	1.61	1.49	1.37	1.29	1.21	1.14	1.08	1.04	1.00	0.96	0.93	0.90	0.88	0.85	0.82	0.80	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61		
27	2.13	1.59	1.43	1.33	1.24	1.16	1.08	1.02	0.97	0.92	0.89	0.86	0.83	0.80	0.77	0.74	0.71	0.69	0.67	0.65	0.63	0.61	0.59	0.57	0.55	0.53	0.51	0.49	0.47	0.45	0.43	0.41	0.39				
28	2.09	1.55	1.42	1.32	1.23	1.14	1.06	1.00	0.95	0.90	0.86	0.82	0.79	0.75	0.72	0.69	0.66	0.63	0.60	0.58	0.55	0.52	0.49	0.46	0.43	0.40	0.38	0.35	0.32	0.30	0.28	0.26	0.24				
29	2.																																				

TABLE I

Single-tailed test		Two-tailed test		
df	α = .05	df	α = .10	
2	3.89	3.05	2.64	2.37
3	3.05	2.52	2.24	2.03
4	2.66	2.34	2.03	1.77
5	2.46	2.08	1.87	1.70
6	2.33	1.97	1.77	1.65
7	2.24	1.90	1.70	1.59
8	2.17	1.84	1.65	1.52
9	2.22	1.79	1.61	1.48
10	2.03	1.76	1.56	1.45
11	2.05	1.73	1.53	1.37
12	2.03	1.70	1.50	1.35
13	2.01	1.68	1.49	1.35
14	1.99	1.67	1.48	1.34
15	1.97	1.65	1.46	1.32
16	1.95	1.63	1.44	1.29
17	1.93	1.61	1.42	1.20
18	1.91	1.59	1.40	1.19
19	1.89	1.57	1.39	1.18
20	1.87	1.55	1.37	1.16
21	1.85	1.53	1.35	1.15
22	1.83	1.51	1.33	1.14
23	1.81	1.49	1.31	1.13
24	1.79	1.47	1.29	1.12
25	1.77	1.45	1.27	1.05
26	1.75	1.43	1.25	1.04
27	1.73	1.41	1.23	1.02
28	1.71	1.39	1.21	1.01
29	1.69	1.37	1.19	0.99
30	1.67	1.35	1.17	0.97
31	1.65	1.33	1.15	0.95
32	1.63	1.31	1.13	0.93
33	1.61	1.29	1.11	0.91
34	1.59	1.27	1.09	0.89
35	1.57	1.25	1.07	0.87
36	1.55	1.23	1.05	0.85
37	1.53	1.21	1.03	0.83
38	1.51	1.19	1.01	0.81
39	1.49	1.17	0.99	0.79
40	1.47	1.15	0.97	0.77
41	1.45	1.13	0.95	0.75
42	1.43	1.11	0.93	0.73
43	1.41	1.09	0.91	0.71
44	1.39	1.07	0.89	0.69
45	1.37	1.05	0.87	0.67
46	1.35	1.03	0.85	0.65
47	1.33	1.01	0.83	0.63
48	1.31	0.99	0.81	0.61
49	1.29	0.97	0.79	0.59
50	1.27	0.95	0.77	0.57
51	1.25	0.93	0.75	0.55
52	1.23	0.91	0.73	0.53
53	1.21	0.89	0.71	0.51
54	1.19	0.87	0.69	0.49
55	1.17	0.85	0.67	0.47
56	1.15	0.83	0.65	0.45
57	1.13	0.81	0.63	0.43
58	1.11	0.79	0.61	0.41
59	1.09	0.77	0.59	0.39
60	1.07	0.75	0.57	0.37
61	1.05	0.73	0.55	0.35
62	1.03	0.71	0.53	0.33
63	1.01	0.69	0.51	0.31
64	0.99	0.67	0.49	0.29
65	0.97	0.65	0.47	0.27
66	0.95	0.63	0.45	0.25
67	0.93	0.61	0.43	0.23
68	0.91	0.59	0.41	0.21
69	0.89	0.57	0.39	0.19
70	0.87	0.55	0.37	0.17
71	0.85	0.53	0.35	0.15
72	0.83	0.51	0.33	0.13
73	0.81	0.49	0.31	0.11
74	0.79	0.47	0.29	0.09
75	0.77	0.45	0.27	0.07
76	0.75	0.43	0.25	0.05
77	0.73	0.41	0.23	0.03
78	0.71	0.39	0.21	0.01
79	0.69	0.37	0.19	-
80	0.67	0.35	0.17	-
81	0.65	0.33	0.15	-
82	0.63	0.31	0.13	-
83	0.61	0.29	0.11	-
84	0.59	0.27	0.09	-
85	0.57	0.25	0.07	-
86	0.55	0.23	0.05	-
87	0.53	0.21	0.03	-
88	0.51	0.19	0.01	-
89	0.49	0.17	-	-
90	0.47	0.15	-	-
91	0.45	0.13	-	-
92	0.43	0.11	-	-
93	0.41	0.09	-	-
94	0.39	0.07	-	-
95	0.37	0.05	-	-
96	0.35	0.03	-	-
97	0.33	0.01	-	-
98	0.31	-	-	-
99	0.29	-	-	-
100	0.27	-	-	-

Value of D =  $\delta/S_p$

TABLE 20

Single-sided test  $\alpha = .05$        $\beta = .50$

**UNCLASSIFIED**

Security Classification

**DOCUMENT CONTROL DATA - R & D**

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) <b>NAD Crane, Indiana</b>		2a. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>
2b. GROUP		
3. REFERENCE TITLE <b>MINIMUM SAMPLE SIZES FOR COMPARISONS USING CONTINUOUS VARIABLES</b>		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)		
5. AUTHOR(S) (First name, middle initial, last name) <b>Lila E. Massa</b>		
6. REPORT DATE <b>8 Oct 1969</b>	7a. TOTAL NO. OF PAGES <b>33</b>	7b. NO. OF REFS
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER (RIS) <b>RDTR No. 159</b>	
b. PROJECT NO.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
c.		
d.		
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11. SUPPLEMENTARY NOTES		12. SECONDO REFERENCE NUMBER AND TITLE
13. ABSTRACT <p>When comparing the means from two populations, minimum sample sizes to detect a given difference may be determined. Presented are tables which may be used when unequal as well as equal sample sizes are desired.</p>		

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
t-test minimum sample sizes precision sample size Lila Massa						

DD FORM NOV. 1968 1473 (BACK)  
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